Quiz 5 (10pts)
Math 214 Section Q1 Winter 2010

Your name:_____ ID#:_____

Please, use the reverse side if needed.

1.(5 pts) Find the length of the curve

$$r = e^{\theta}, \quad 0 \le \theta \le \pi/2.$$

Solution.

$$L = \int_0^{\pi/2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta = \int_0^{\pi/2} \sqrt{e^{2\theta} + e^{2\theta}}\, d\theta$$
$$= \sqrt{2} \int_0^{\pi/2} e^{\theta} d\theta = \sqrt{2} e^{\theta} \Big|_0^{\pi/2} = \sqrt{2}\left(e^{\pi/2} - 1\right).$$

2.(5 pts) Find the area inside the curve $r = 2 + \cos\theta$.

Solution.

$$A = \frac{1}{2}\int_0^{2\pi} r^2(\theta) d\theta = \frac{1}{2}\int_0^{2\pi} (2+\cos\theta)^2 d\theta = \frac{1}{2}\int_0^{2\pi} (4+4\cos\theta+\cos^2\theta) d\theta$$
$$= \frac{1}{2}\int_0^{2\pi} (4+4\cos\theta+\frac{1}{2}+\frac{1}{2}\cos 2\theta) d\theta = \frac{1}{2}(4\theta+4\sin\theta+\frac{1}{2}\theta+\frac{1}{4}\sin 2\theta)\Big|_0^{2\pi}$$
$$= \frac{1}{2}(8\pi + \pi) = \frac{9\pi}{2}.$$